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Iván Werning
ⓡ
Guido Lorenzoni
ⓡ
Veronica Guerrieri

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ABSTRACT

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Iván Werning
Massachusetts Institute of Technology
Department of Economics
and NBER
iwerning@mit.edu

ⓡ

Guido Lorenzoni
University of Chicago
Booth School of Business
and NBER
Guido.Lorenzoni@chicagobooth.edu

ⓡ

Veronica Guerrieri
University of Chicago
Booth School of Business
and NBER
vguerrie@chicagobooth.edu

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1 Introduction

"We may find ourselves in the challenging scenario in which our dual-mandate goals are in tension."

Jerome H. Powell (Federal Reserve Chair, Speech at the Economic Club of Chicago, April 16, 2025)

A central bank confronted with a significant shift toward protectionist policies faces two immediate risks. First, tariffs exert inflationary pressure by directly raising the cost of imported goods—affecting both final consumer products and intermediate inputs used by domestic producers. Second, by distorting consumption and production decisions, tariffs generate efficiency losses that lower the economy's potential output. Together, these effects amount to a "supply shock" that raise the specter of slower growth alongside rising inflation—in other words, a stagflation scenario.

In this paper, we formalize this intuition and shed further light on existing results using a simple model that highlights how tariffs distort the use of intermediate inputs—highly relevant in a world with large volumes of trade in intermediate goods and highly integrated supply chains. Our analysis yields a sharp conclusion: a tariff in an open economy is exactly equivalent to a labor wedge shock in a closed economy. In effect, our equivalence result shows that a tariff can be studied *as if* it were a cost-push shock in a standard New Keynesian (closed economy) model.

One benefit of our result is that cost-push shocks—such as those that shift the Phillips curve upward—are well studied and understood. They force monetary policy to navigate a trade-off between its dual objectives.[1] Viewed through the lens of our result, tariffs introduce a wedge-type distortion that creates an unavoidable conflict for monetary policy. The set of feasible inflation-output combinations deteriorates, leaving a central bank unable to simultaneously achieve both price stability and natural output.

Behind our result lie two distinct effects of tariffs: (i) a fall in labor productivity, and (ii) a sharper fall in firm profitability and, consequently, in the natural wage consistent with zero inflation. It is this discrepancy that generates the positive labor wedge—or cost-push shock. In fact, the decline in productivity turns out to be zero to a first-order approximation, while the decline in the required wage is not. As a result, the labor wedge has a first-order impact on the economy, whereas the drop in productivity—perhaps surprisingly—does not.[2]

Is a tariff expansionary or contractionary? The answer depends on how monetary policy responds, underscoring the value of an optimal policy framework.[3] Based on our equivalence result, a tariff is best understood as a supply-side disturbance—not a demand-side disturbance—that introduces a trade-off for monetary policy. If the central bank prioritizes price stability, the tariff will lead to a sharp short-run contraction. Alternatively, the central bank can offset this contraction by pursuing an expansionary response. In sum, the notion of a "neutral" monetary policy is not well defined. This highlights the central importance of understanding the optimal policy response.

Having established that a tariff acts as a labor wedge shock, we characterize the welfare-

[1] It is sometimes less appreciated, however, that not all "supply shocks" take the same form. For example, shocks to productivity do not necessarily create the same tension between central bank objectives as do "wedge" shocks. Indeed, we show that tariffs create a drop in productivity and a labor wedge, but only the latter shapes the monetary policy response.

[2] Of course, the second-order decline in productivity is still welfare-relevant and has economic effects that merit policy attention. Fortunately, the impact of these productivity shocks is straightforward to analyze. However, productivity shocks do not generate the same policy trade-offs as labor wedge shocks.

[3] The traditional beggar-thy-neighbor view of older Keynesian models saw unilateral tariffs as having an expansionary effect on the domestic economy, due to their expenditure-switching effect, moving domestic demand away from foreign goods and towards domestic goods. Krugman (1982) questioned the direction of this effect and explored the potential contractionary effect of tariffs through the exchange rate. In recent years, there is a growing literature that uses new Keynesian models to analyze the effect of tariffs. This literature has also pointed out that tariffs can be contractionary. This is a central point of Barattieri et al. (2021) who also show empirically the contractionary effects of tariff shocks.

maximizing monetary policy response. We begin with the textbook case of price rigidity and flexible wages, then extend the analysis to include nominal wage rigidity. In both settings, optimal policy allows inflation to rise temporarily and keeps output above its natural (flexible-price) level in the short run. This inflation arises precisely because output is held above the new, lower natural level, which falls due to the efficiency loss induced by the tariff. Over time, output gradually declines toward the distorted steady state. Maintaining zero inflation would require an abrupt drop in activity—an outcome optimal policy avoids by smoothing the adjustment path. Consumption and labor even rise modestly on impact, but both eventually contract. Thanks to the tractability offered by our equivalence result and our continuous-time formulation, we are able to derive all these results analytically. When wages are sticky, the cost of disinflation rises, and the central bank has less flexibility to engineer a smooth adjustment—though the core logic of the optimal response remains unchanged.

The backbone of our model is relatively standard, but to obtain our sharp equivalence results, we make a few important modeling choices. Specifically, we study a small open economy that takes world prices as given and is subject to a trade balance constraint. These simplifying assumptions allow us to focus squarely on the core trade-off between output stabilization and inflation control. In particular, the small open economy assumption abstracts from terms-of-trade manipulation, directing attention to more traditional monetary policy concerns. Of course, terms-of-trade effects are important in their own right and have been extensively studied in both trade and international macroeconomics. The trade balance assumption plays a role in delivering our exact equivalence result, but we believe the result remains a good approximation for monetary policy responses even when trade balance is not imposed. On balance, we view these assumptions as a worthwhile trade-off to gain clarity and insight into a central economic policy question.

Our paper contributes to the growing literature on the implications of tariffs for monetary policy including Barattieri et al. (2021), Jeanne (2021), Bergin and Corsetti (2022), Auray et al. (2024), Cuba-Borda et al. (2024) Bianchi and Coulibaly (2025), Monacelli (2025), Sebnem Kalemli-Ozcan (2025) and Auclert et al. (2025). The closest papers that also focus on optimal monetary policy responses are Bianchi and Coulibaly (2025) and Monacelli (2025). These papers provide numerical results based on model simulations and agree on a core result: a short-run increase in inflation, and output above its natural level, are part of the optimal monetary policy response to a tariff shock.[4] Our results agree with this conclusion and shed further economic light on the positive mechanisms and normative

[4]Bergin and Corsetti (2022) obtain somewhat different results, but study a different model and policy experiments, including the effects of a retaliatory trade war and coordinated monetary policy.

forces behind it by providing analytical results that characterize generally the optimal policy response. In terms of interpretation, Bianchi and Coulibaly (2025) focus on the fiscal revenue coming from the tariff and on the fact that consumers buying foreign goods do not internalize the effect on their purchases on fiscal revenue. This fiscal externality is the reason why monetary policy may choose a more inflationary path. Our interpretation of the forces at play is more in line with traditional analysis of monetary policy that focuses on the trade-off between output gaps and inflation. In doing so, we trace a direct connection between our results and the classic analysis of optimal monetary policy with a distorted steady state of Benigno and Woodford (2004), since the tariff induces a new distorted steady state.

2 Tariffs as Cost-Push Shocks in Static Economy

We begin with a simple single-period neoclassical economy without nominal rigidities. This setup allows us to isolate two inefficiencies arising from the imposition of tariffs. In the sections that follow, we embed this static framework into a dynamic setting with nominal rigidities. The inefficiencies identified here will then serve as driving shocks in that framework.

The main result of this section is that a tariff is equivalent to two distortions: (i) a fall in productivity and (ii) a rise in the labor wedge (akin to a tax). Although both distortions are present and affect welfare, perhaps surprisingly, only the labor wedge matters to a first-order approximation.

2.1 Primitives: Technology, Utility and Trade

The Home economy consumes and produces a single final good. A representative agent has utility function

$$U(C, L),$$

assumed concave, differentiable, increasing in C and decreasing in L.

Production combines labor with an imported intermediate input by

$$Y = F(L, M),$$

where F is concave, differentiable and has constant returns to scale. Here Y denotes the final output, L labor and M the imported intermediate input.[5]

[5] The production function $F(L, M)$ describes the efficient final output given (L, M) after possibly op-

The Home country exports this domestic good to foreign countries. Letting X denote exports, we then have the feasibility condition

$$Y = C + X,$$

where C denotes consumption.

The Home country takes world prices as given and must satisfy trade balance.[6] We normalize prices $P_Y = P_M = 1$ so the trade balance condition becomes $X = M$. Domestic consumption then satisfies $C = F(L, M) - M$, which we can write as

$$C = A(\mu)L \tag{1}$$

where

$$A(\mu) \equiv F(1, \mu) - \mu$$

and $\mu \equiv M/L$ represents the input-to-labor ratio. Equation (1) indicates that for any given ratio μ we have a linear production possibility frontier (PPF) between labor and consumption with $A(\mu)$ acting as a productivity parameter.

2.2 Equilibrium with and without Tariffs

We now study the equilibrium and its dependence on tariffs, starting with the benchmark without tariffs.

No Tariff, First-Best Equilibrium. Without a tariff, the competitive equilibrium outcome coincides with the first best allocation. The first-best allocation maximizes $U(C, L)$ subject to (1). One can solve this allocation in two steps. First, we maximize the input-to-labor ratio to achieve the highest productivity

$$A^* = \max_{\mu} A(\mu)$$

Second, we choose consumption and labor to maximize utility subject to this maximal linear frontier $C = A^* L$.

timizing over many other production choices, such as the production and use of domestic inputs. For example, consider a roundabout domestic input, based on the final good, then $Y + y = \hat{F}(L, M, y)$ then $F(L, M) = \max_y \{\hat{F}(L, M, y) - y\}$. We can similarly collapse an economy with multiple sectors that produce various inputs from labor and each other into the final good production function $F(L, M)$.

[6] We set initial net foreign assets to zero $NFA_0 = 0$.

Equilibrium with Tariffs. We now study the distorted competitive equilibrium when the domestic economy imposes a tariff τ on its imports. We begin with firms and then turn to households.

Consider the firm problem. Firms choose their input-to-labor ratio $\mu = M/L$ to maximize $A(\mu) - \tau\mu$ yielding $\mu(\tau)$. Second, firms can also optimize over labor taking as given a real wage $W(\tau)$. At an interior equilibrium, due to constant returns to scale, this gives zero profits so the wage is given by

$$W(\tau) = \max_{\mu}\{A(\mu) - \tau\mu\} = A(\mu(\tau)) - \tau\mu(\tau). \tag{2}$$

Economically, the dependence of the wage on the tariff represents moving along the iso-cost curve for the firm: if the tariff rises, then the wage must fall to compensate firms so that they are still willing to produce at a unit price.

Evaluating the possibility frontier (1) with the equilibrium ratio $\mu(\tau)$ gives

$$C = A(\mu(\tau))L, \tag{3}$$

a linear production possibility frontier with $A(\mu(\tau))$ acting as a productivity parameter.

Turning to households, they maximize their utility subject to a budget constraint

$$C = W(\tau)L + T$$

where T is a lump sum transfer that households take as given, equal, in equilibrium, to the revenue from the tariff: $T = \tau M$. The first-order optimality condition is

$$-\frac{U_L(C,L)}{U_C(C,L)} = W(\tau). \tag{4}$$

Putting firms and households together, given the constructed productivity $A(\mu(\tau))$ and wage $W(\tau)$, an equilibrium is entirely characterized by two equations in two unknowns: the optimal labor supply condition (4) and the production possibility frontier (3).

2.3 Two Inefficiencies from Tariffs

A positive tariff $\tau > 0$ introduces two sources of inefficiency relative to the first best.

First, the tariff introduces productive inefficiency. This occurs because firms maximize $A(\mu) - \tau\mu$ rather than $A(\mu)$. As a result, for $\tau \neq 0$, we have $A(\mu(\tau)) < A^*$, whereas for $\tau = 0$ we have $A(\mu(0)) = A^*$. The tariff distorts the use of the imported input, reducing the level of consumption that can be achieved for any given level of labor L.

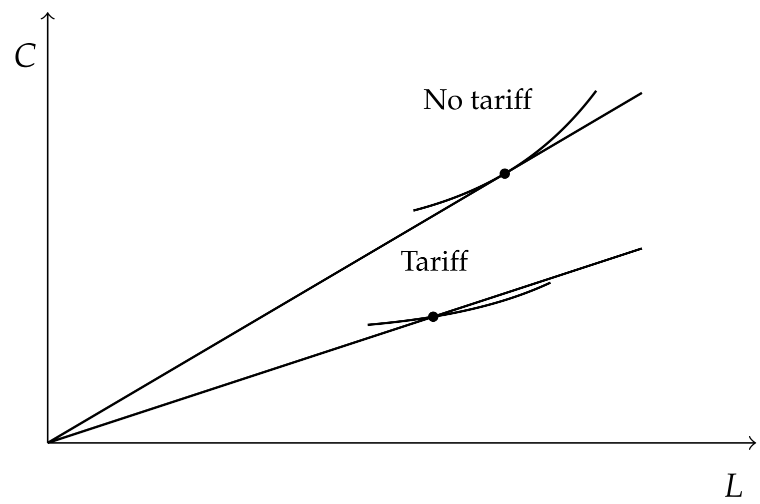

Figure 1: Equilibrium with and without a tariff

This represents a downward shift in the linear production possibility frontier (3).

Second, the tariff lowers the wage, so that

$$W(\tau) < W(0)$$

for $\tau > 0$, since we have

$$W'(\tau) = -\mu(\tau) < 0 \tag{5}$$

by the Envelope Theorem applied to equation (2). As mentioned above, the lower wage reflects reduced firm profitability under a higher τ.

Finally, an additional distortion arises from the fact that the wage no longer equals productivity, so that $W(\tau) \neq A(\mu(\tau))$ for $\tau \neq 0$. The resulting labor wedge is given by

$$A(\mu(\tau)) - W(\tau) = \tau\mu(\tau) \geq 0,$$

which follows directly from (2). Note that the labor wedge is strictly positive unless the tariff is zero ($\tau = 0$) or so prohibitively high that it induces zero imports ($\mu(\tau) = 0$).

These two effects are illustrated in Figure (1). The two straight lines represent the linear possibility frontier $C = A(\mu(\tau))L$, with and without the tariff. We also plot consumer indifference curves tangent to the wage $W(\tau)$ in each case. Without a tariff $A(0) = W(0)$ so the household choses the optimum along the frontier at the point of tangency. With the tariff, $W(0) < A(0)$ and the equilibrium is no longer tangent to the frontier; labor is inefficiency reduced.

If we consider small tariffs starting from free trade then the production inefficiency is initially small. Formally, it is zero to a first order. To see this, note that from the definition of $A(\mu)$ we have

$$A'(\mu(0)) = 0 \tag{6}$$

and thus $A'(\mu(0))\mu'(0) = 0$.

On the other hand, the tariff does have a first-order effect on the wage and the labor wedge. To see this note that (6) implies that $W'(0) = -\mu(0) < 0$ so that $A'(\mu(0))\mu'(0) - W'(0) = \mu(0) > 0$.

Since, to first order approximation, productivity is unaffected while the labor wedge is, this represents a (local) pure substitution effect that reduces both labor and consumption. Letting $L(\tau)$ denote equilibrium labor, we have $L'(0) < 0$.

We summarize these results in the next proposition.

Proposition 1. *A tariff $\tau > 0$ leads to lower productivity, a lower wage, and a higher labor wedge: $A(\mu(\tau))$ and $W(\tau)$ are both decreasing, while $A(\mu(\tau)) - W(\tau)$ is increasing. The productivity effect is zero to a first order, but the the effects on the wage and labor wedge are non zero. Labor is affected negatively to a first order.*

The economic consequences of this characterization go beyond its static neoclassical setting and play out each period in a dynamic economy with nominal price and wage rigidities.

2.4 Quadratic Approximation to Welfare

We now briefly discuss the effects of the tariff on welfare. Let $\mathcal{U}(\tau)$ denote household utility as a function of the tariff. As usual, starting from a zero tariff, the first-order effect on household welfare is zero: $\mathcal{U}'(0) = 0$. Since we know that $\mathcal{U}(\tau) \leq \mathcal{U}(0)$, it follows that $\mathcal{U}''(0) \leq 0$. Indeed, computing the second-order effects, we find a utility loss (expressed in consumption units) of

$$\frac{\mathcal{U}''(0)}{U_C} = L(0)A''(\mu(0))\mu'(0)^2 - W'(0)L'(0) < 0, \tag{7}$$

where U_C is the marginal utility of consumption at the zero tariff allocation.[7]

The first term captures the direct effect from the loss in productive efficiency due to the distortion in the choice of the ratio μ. The second term captures the indirect distortionary

[7] See the appendix for a derivation where it is also shown that $\mathcal{U}''(0) = U_C \cdot [L(0)\mu'(0) + \mu(0)L'(0)]$.

effect of the labor wage on labor. Both terms are negative, so both sources of inefficiency contribute to the reduction in welfare.

A similar expression to (7) reemerges in the next section, when we analyze optimal monetary policy using a second-order approximation of the welfare function.

2.5 Towards Adding Nominal Rigidities

We briefly explore an alternative approach to deriving the results above. This method is particularly useful when prices do not fully adjust and firms produce to meet demand at a given price. At this stage, what follows is best interpreted as a partial equilibrium analysis.

For any given output level Y, firms must minimize costs $WL + (1+\tau)M$ subject to $F(L, M) = Y$. This leads to the familiar condition

$$\frac{F_M(1,\mu)}{F_L(1,\mu)} = \frac{W}{1+\tau},$$

with solution $\hat{\mu}(\frac{1+\tau}{W})$. We see that both τ and W influence the choice of μ. Economically, this reflects the fact that it is the relative price of the two inputs $(1+\tau)/W$ that matters and that we are considering any wage W because we are not constraining ourselves to the iso-cost curve as before.

Previously, the wage was endogenous and pinned down by the iso-cost condition, to ensure the firm was willing to produce at a unit price. With the present formulation, this amounts to evaluating $\mu(\tau) = \hat{\mu}(\frac{1+\tau}{W(\tau)})$.[8]

This cost-minimization formulation with the ratio $\hat{\mu}(\frac{1+\tau}{W})$ will be useful in the next section where we consider firms that, due to price rigidities, do not optimize over output for a given price, but instead produce to meet demand. In that economy, the wage is also endogenous—determined by households—and is influenced by monetary policy and potentially subject to nominal rigidities. For both reasons, the real wage does not necessarily equal the flexible equilibrium real wage found earlier $W(\tau)$. Thus, we will be using an analog of $\hat{\mu}(\frac{1+\tau}{W(\tau)})$ rather than with $\mu(\tau)$.

[8]The wage itself solves

$$W(\tau) = A\left(\hat{\mu}\left(\frac{1+\tau}{W(\tau)}\right)\right) + \tau\hat{\mu}\left(\frac{1+\tau}{W(\tau)}\right).$$

But we saw it was more directly obtained from $W(\tau) = \max_{\mu}\{A(\mu) + \tau\mu$.

3 Dealing with Cost-Push Shocks in a Dynamic Economy

We now extend our simple economy to a dynamic setting with nominal rigidities. We begin with constraints on price changes and later incorporate wage rigidities.

With flexible prices, there is nothing the monetary authority can do to offset the inefficient decline in output. However, nominal rigidities allow the central bank to keep output and consumption temporarily above their flexible-price levels—that is, to smooth the transition to a lower level of activity. This can be beneficial, as it keeps both labor and intermediate inputs temporarily closer to their efficient levels. As we shall see in this section, this comes at the cost of higher inflation, so an optimizing central bank must balance these two considerations.

3.1 The Dynamic Model

The economy operates in continuous time $t \geq 0$. Households have utility

$$\int_0^\infty e^{-\rho} U(C_t, L_t)\, dt,$$

with $\rho > 0$. We now specialize the utility function to

$$U(C, L) = \log C - \frac{1}{1+\phi} L^{1+\phi}.$$

The domestic final good is produced competitively by combining a continuum of varieties

$$Y_t = \int_0^1 \left(Y_{it}^{1-\frac{1}{\varepsilon}} di \right)^{\frac{1}{1-\frac{1}{\varepsilon}}} di,$$

where Y_{it} is variety i. Each variety i is produced by a monopolistically competitive with the production function

$$Y_{it} = F(L_{it}, M_{it}) = \left(a_L^{\frac{1}{\eta}} L_{it}^{1-\frac{1}{\eta}} + a_M^{\frac{1}{\eta}} M_{it}^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-\frac{1}{\eta}}},$$

where η is the constant elasticity of substitution between labor and the imported input.

We retain our simplifying assumptions with regards to international trade. The relative price of the final good relative to the imported input is given and independent of domestic policy.[9] Moreover, we assume closed financial markets so that trade balance

[9]Thus, our economy is a limit case of the open-economy model of Gali and Monacelli (2005), when all country goods become perfect substitutes.

holds in all periods. The resource constraint for the country is again

$$C_t = Y_t - M_t.$$

Each monopolistic competitor producer faces a downward sloping demand schedule $Y_{it} = (p_{it}/P_t)^{-\epsilon} Y_t$ for their variety. They reset their price at random intervals with Poisson arrival rate $\theta > 0$. They always meet demand.

At time $t = 0$, a positive and permanent tariff τ is introduced unexpectedly (an "MIT shock").[10] As in the previous section, the tariff alters the quantities and relative prices that would prevail under flexible prices—in exactly the same way, in each period, as in the static model. We refer to this as the flexible-price allocation. However, due to nominal rigidities, monetary policy is non-neutral and can affect the equilibrium allocation. In particular, the flexible-price allocation is attainable, but other allocations that deviate from it are as well.

As is common in this class of models, we introduce a subsidy on production that offsets the monopolistic markup distortion. This ensures that the initial steady state—absent shocks—coincides with the first-best allocation, allowing us to focus on fluctuations and stabilization policy.

3.2 Equilibrium and Welfare

The driving force behind inflation in New Keynesian models is the gap between nominal marginal costs of producers and the price of output (adjusted for desired markups). When this gap is positive, firms have an incentive to raise prices, and inflation follows. Using lowercase variables to denote log deviations from the initial steady state, nominal marginal costs are simply

$$s_L w_t + s_M (p_t^M + \tau),$$

where w_t is the nominal wage, p_t^M is the price of imported inputs in domestic currency, τ is the tariff, and s_L and s_M are the shares of the labor input and the imported intermediate good in production costs in the initial steady state, with $s_L + s_M = 1$.[11]

Because of fixed terms of trade $p_t^M = p_t$ and the gap between nominal marginal costs

[10] The permanence of the tariff is not crucial to any of our arguments; we adopt it as a benchmark because it helps highlight the temporary nature of the optimal response. In particular, our characterization in terms of a cost-push shock applies to any sequence of tariffs $\{\tau_t\}$, which would then imply a corresponding sequence of cost-push shocks.

[11] With a slight abuse of notation in all log-linearized expressions we use τ to denote $\ln(1+\tau)$.

and the domestic price p_t is

$$s_L w_t + s_M(p_t^M + \tau) - p_t = s_L \omega_t + s_M \tau,$$

where ω_t denotes the real wage $w_t - p_t$.

Following standard derivations we can then obtain the Phillips curve

$$\rho \pi_t = \theta(\rho + \theta)(s_L \omega_t + s_M \tau) + \dot{\pi}_t \tag{8}$$

where π_t is inflation, ρ is the consumer discount factor, and θ is the arrival rate of the Calvo process.

The direct effect of the tariff on inflation is captured by the term $s_M \tau$. The model intuition is in line with the standard narrative: a tariff increases costs for the firms using imported intermediates; for a given real wage ω_t, these cost pressures translates into nominal price increases.

To complete the equilibrium characterization we need four additional conditions. The labor supply condition still takes the form (4) and, after some substitutions, we have

$$(1 + \phi) l_t = \omega_t. \tag{9}$$

The use of imported intermediates depends on the relative price of the two inputs, labor and imported intermediate goods, and is given by the demand function

$$m_t = l_t + \eta(\omega_t - \tau). \tag{10}$$

Finally, output and consumption are given by

$$y_t = s_L l_t + s_M m_t, \tag{11}$$

and

$$c_t = l_t. \tag{12}$$

This last equation is the log-linear approximation of the production frontier $C_t = A_t L_t$, which is the same as (3) in the previous section. The linearized form (12) mirrors our observation in Section (2) that the production frontier is unaffected by the tariff shock and by monetary policy to a first order.

In contrast, the effects of the tariff and of monetary policy do appear once we turn to the second-order approximation of welfare,

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\frac{1}{\eta} s_M (m_t - l_t)^2 + (1 + \phi) l_t^2 + \Psi_P \pi_t^2 \right] dt, \tag{13}$$

with

$$\Psi_P = \frac{\varepsilon}{\theta(\rho+\theta)}.$$

The first two terms in this welfare function directly reflect the two inefficiencies identified in the previous section and derived formally in equation (7). The first term captures productive inefficiency arising from a distorted input-to-labor ratio $m_t - l_t \neq 0$. The second term is the inefficiency in employment. With balanced growth preferences, the efficient level of l_t is equal to 0 regardless of productivity, which explains why this term is simply proportional to l_t^2; the natural rate of employment is constant. Finally, the last term reflects the cost of nonzero inflation, which is standard in models with nominal rigidities. It arises from the inefficient price dispersion that inflation induces across varieties.

3.3 The Phillips Curve and Policy Trade-Offs

After some substitutions, the Phillips curve (8) can be rewritten compactly in the traditional form

$$\rho\pi_t = \kappa y_t + \epsilon + \dot{\pi}_t, \tag{14}$$

where the term ϵ is proportional to τ.[12] A positive tariff $\tau > 0$ generates a supply shock $\epsilon > 0$ in equation (14); that is, it shifts of the Phillips curve upward.

To keep inflation at zero, the central bank must allow output contract in order to set the expression $\kappa y_t + \epsilon$ to zero. The required output contraction corresponds to the flexible-price allocation discussed in Section (2). Using a tilde to denote flexible-price allocations, we need

$$y_t = \tilde{y} \equiv -\left(\frac{1}{1+\phi} + \eta\right)\frac{s_M}{s_L}\tau < 0.$$

There are two underlying forces causing $\tilde{y} < 0$, which, once again, reflect the two inefficiencies identified in the previous section: a reduction in the input-to-labor ratio, $m_t - l_t$, and a reduction in labor, l_t.

Surprisingly, the welfare function (13) can also be rewritten in its traditional form

$$-\frac{1}{2}\int_0^\infty e^{-\rho t}\left(\Psi_Y y_t^2 + \Psi_P \pi_t^2\right) dt. \tag{15}$$

[12] The expressions for κ and ϵ are

$$\kappa = \theta(\rho+\theta)\frac{s_L(1+\phi)}{1+\eta s_M(1+\phi)},$$
$$\epsilon = \theta(\rho+\theta)\frac{1+\eta(1+\phi)}{1+\eta s_M(1+\phi)} s_M \tau.$$

This expression, together with the Phillips curve (14), highlights the core tension in the model. The level of output that maximizes welfare remains at the initial steady state, $y_t = 0$. However, the level of output consistent with zero inflation is now $\tilde{y} < 0$. The central bank can no longer achieve both objectives simultaneously.

Intuitively, optimal policy manages this trade-off by keeping output temporarily above its flexible-price level immediately after the tariff, and then gradually adjusting toward it. Doing so comes at the cost of some positive inflation. The general characterization of the optimal policy is given in the next proposition.

Proposition 2. *The optimal policy following the imposition of a permanent tariff $\tau > 0$ yields an equilibrium allocation with output given by*

$$y_t = \left(1 - e^{-rt}\right) \tilde{y}, \tag{16}$$

and implies:

(i) unchanged output on impact, $y_0 = 0$, followed by decreasing output converging to its new, lower steady state: $y_t \to \tilde{y} < 0$;

(ii) an increase in employment on impact $l_0 > 0$, followed by decline in employment toward its new, lower steady state: $l_t \to \tilde{l} < 0$;

(iii) an increase in inflation on impact, $\pi_0 > 0$, followed by a gradual return to zero: $\pi_t \to 0$.

The coefficient r in equation (16), which governs the speed of adjustment of y_t, is defined in the appendix.

The proposition is illustrated in Figure 2 for a numerical example. The unit of time is a year and parameters are set to

$$\rho = 0.03, \quad \phi = 1, \quad s_M = 0.1, \quad \eta = 1, \quad \varepsilon = 2, \quad \theta = 1.$$

The tariff shock is set to $\tau = 0.20$. The figure shows the responses of real variables and inflation following the tariff shock.

The result that the central bank only temporarily keeps y_t above $\tilde{y}$ only temporarily is a classic feature of economies with a distorted steady state (Benigno and Woodford, 2004). In this case, the initial steady state is undistorted, but the new steady state—following the permanent tariff—is distorted. The key intuition is that maintaining output above its flexible-price level far into the future imposes inflationary distortions in all prior periods, as price-setters anticipate the impact of future policy on marginal costs. As the horizon extends, the cumulative inflation costs from earlier periods grow, eventually outweighing the short-term benefits of boosting output. This makes it too costly for the central bank to sustain such a policy indefinitely.

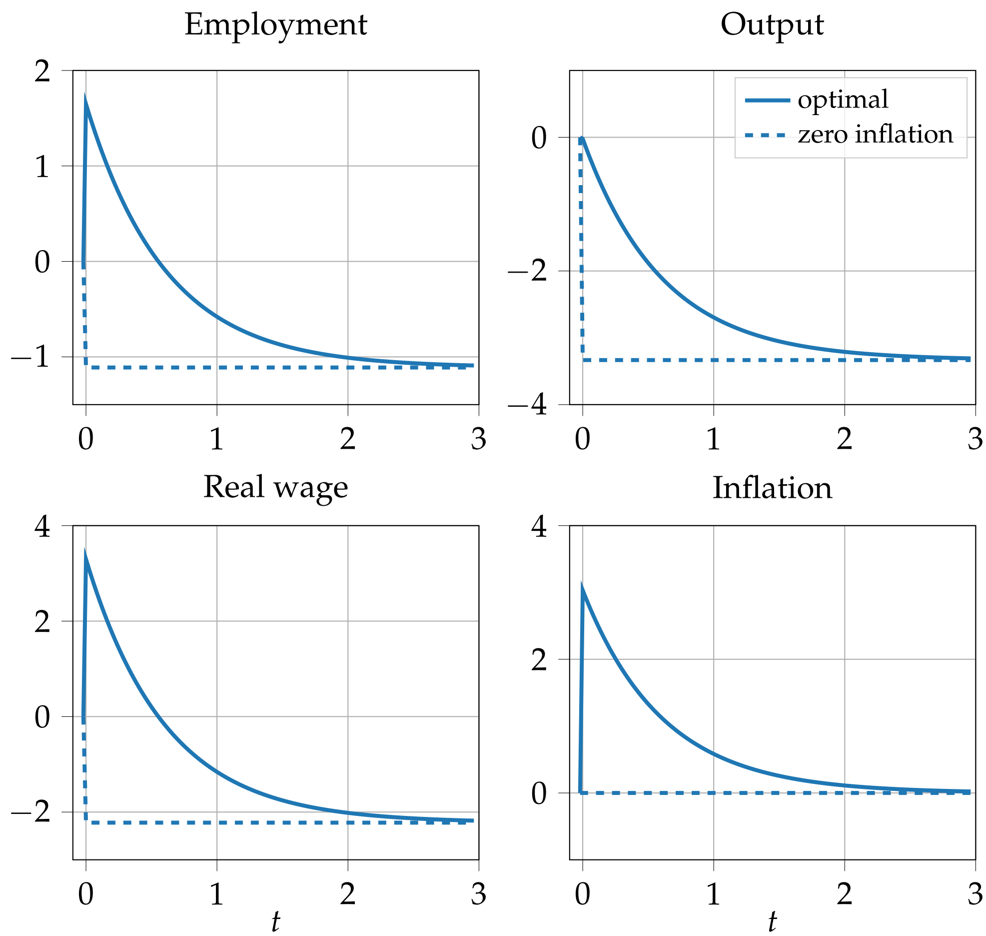

Figure 2: Responses to the tariff shock, under optimal policy and zero-inflation policy
All variables in log deviations from steady state (times 100)

The result that output starts exactly at its initial steady state $y_0 = 0$ reflects the fact that, in continuous time, the inflationary costs are negligible over an arbitrarily short interval following the shock. This result would not be apparent in discrete time, although it would be approached for sufficiently small time periods.

Maintaining output unchanged on impact requires a temporary increase in employment above its initial steady state to compensate for the reduction in imported inputs caused by the tariff. In this model, raising employment above its initial level improves efficiency because it increases the real wage ω_t. Since the input-to-labor ratio, $m_t - l_t$, is increasing in $\omega_t - \tau$, keeping real wages elevated partially offsets the inefficient reduction in $m_t - l_t$ induced by the tariff.

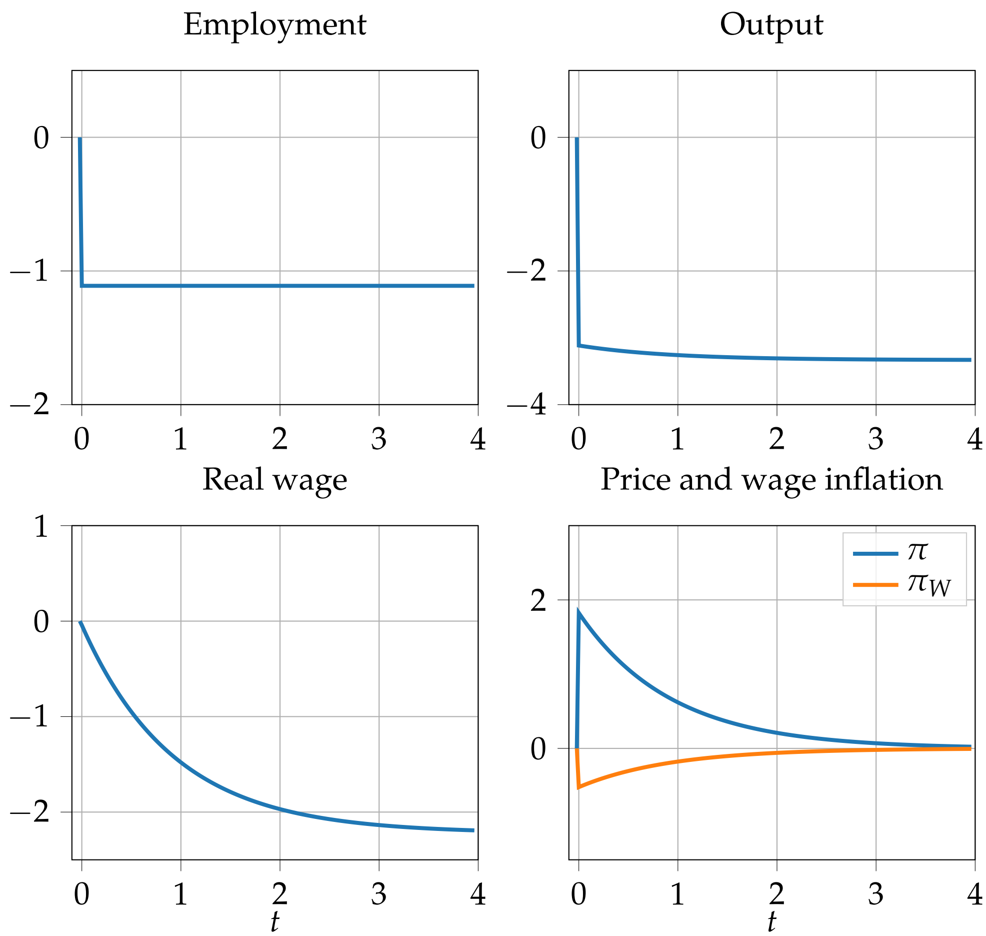

Figure 3: Sticky wage model, with employment at its flexible-price level
All variables in log deviations from steady state (times 100)

3.4 Nominal Wage Rigidities

In the model so far—with sticky prices and flexible wages—real wage adjustment can occur instantly and without cost. Moreover, a zero-inflation outcome can be achieved by replicating the flexible-price allocation in quantities, even though this path is suboptimal. Inflation, in this case, results from the central bank's choice to smooth the economy's adjustment toward the more distorted equilibrium.

Whether the so-called "divine coincidence" holds in this context is a matter of semantics. If we take the flexible-price allocation as the reference point, then divine coincidence holds, since the central bank can achieve that allocation while maintaining zero inflation. If, instead, we take the efficient allocation $y = 0$ as the reference, divine coincidence fails. What is unambiguous is that a trade-off exists between the central bank's welfare objectives.

Figure 4: Sitcky wage model, achieving a lower inflation path
All variables in log deviations from steady state (times 100)

If we add sticky wages, real wage adjustment can no longer occur without some combination of wage deflation and/or price inflation, both of which introduce distortions. Moreover, even if the central bank replicates the quantities from the flexible-price allocation, inflation still arises—unlike in the previous model. This is because choosing $l_t = \tilde{l}$ implies that nominal wages adjust only gradually. As a result, nominal marginal costs exceed firms' prices along the transition path, generating price inflation. The details of this argument are provided in the appendix. An example is illustrated in Figure (3).

The welfare function now takes the following form

$$-\frac{1}{2}\int_0^\infty e^{-\rho t}\left[\eta s_M\left(\omega_t-\tau\right)^2+(1+\phi)\,l_t^2+\Psi_\pi\pi_t^2+\Psi_w\pi_{Wt}^2\right]dt, \tag{17}$$

where ω_t is now a state variable.

Achieving a lower rate of price inflation than the one shown in Figure (3) requires a contraction in employment larger than $\tilde{l}$, along with greater wage deflation. This is the only way to generate the reduction in real wages needed to offset the price pressure from imported inputs. This response is illustrated in Figure (4), which shows a contraction in employment and output that overshoots (from below) the level needed to achieve zero inflation in the long run.

The first result here is that, with sticky wages, containing the inflationary effects of the tariff shock is more costly, as it requires a contraction in real activity larger than under flexible wages.

However, the contraction shown in Figure (4) is not the optimal response. As in the case with flexible wages, optimal policy moves in the opposite direction: it entails a smoother path for quantities, keeping the economy temporarily above its flexible-price allocation and gradually converging to lower activity. We illustrate this in the example in Figure 5 and prove the result in Proposition (3) below.

While the two cases—flexible wages and sticky wages—are similar in requiring a gradual adjustment of quantities, there is one qualitative difference, visible when comparing Figures 2 and 5. In the first model, we observe stable output and higher employment on impact,$y_0 = 0, l_0 > 0$; in the second, we observe stable employment and lower output, $l_0 = 0, y_0 < 0$. The input-to-labor ratio chosen by cost-minimizing firms is a function of $\omega_t - \tau$. The difference then arises because, with nominal wage rigidities, the real wage cannot be influenced by monetary policy in the very short run—the real wage becomes a state variable in the model.

In sum, nominal wage rigidities make contractionary policy more costly as a tool for reducing inflation, but they also reduce the gains from keeping the economy above its flexible-price benchmark.

Proposition 3. *With both sticky prices and sticky wages, optimal policy in response to a permanent tariff $\tau > 0$ yields an equilibrium with:*

(i) a fall in output on impact, $y_0 < 0$, followed by a larger output decline in the long run: $y_t \to \tilde{y} < y_0 < 0$;

(ii) unchanged employment on impact, $l_0 = 0$, and lower employment in the long run: $l_t \to \tilde{l} < 0$;

(iii) positive average price inflation along the transition, $\int_0^\infty \pi_t dt > 0$, with inflation converging to zero in the long run: $\pi_t \to 0$.

Unlike in Proposition 2, we do not have a closed-form characterization of the transition path for inflation. However, we can still show that inflation is, on average, positive along the transition path.

Figure 5: Sticky wage model, optimal policy
All variables in log deviations from steady state (times 100)

Whether wage inflation is positive at the optimum is ambiguous and generally depends on the model parameters. In particular, it reflects a trade-off between the welfare costs of sustained wage and price inflation and the benefits of keeping quantities closer to their efficient levels for longer. In the example shown in Figure 5, both price inflation and wage inflation are positive.

4 Connections to Policy Debate

We now discuss and extend the results above, drawing more connections to the policy debate.

Commitment. A prominent concern of central bankers is the problem of commitment. In the current situation, coming from a period of high inflation, the concern is especially salient in policy discussions. Does our simple model provide any guidance on the issue of commitment?

First, let us clarify that all the optimal policy paths derived above were obtained under the assumption that the central bank commits to the entire path for the real variables l_t and y_t. In general, the policy tension in the objective function becomes a bigger problem when the central bank acts under discretion and the outcome depends on what assumptions we make about the central bank's reputation.

Second, models with a distorted steady state imply that the central bank does not want to exploit the tradeoff between output and inflation in the long run. We see this general result applied in all the simulations above that show real variables converging to their flexible-price level, even though that level is now inefficient. This is an observation that is often invoked to advocate for policy rules that ignore distortions in the economy caused by market power, the tax system and so on. In other words, if we are in a stable environment monetary policy should not try systematically try to expand the economy even if the equilibrium level of activity is distorted below its first-best level by various wedges. The approach advocated by Woodford to deal with this issue is to design policy from a "timeless perspective".[13] Imagine a central bank that can commit to policy in the past, before any tariff shock. Would that central bank respond to the tariff shock as in the optimal plots above, or would it have preferred to commit to keeping inflation lower?

A somewhat counterintuitive result is that the plots above are part of an optimal plan also from the timeless perspective, as long as the probability of the tariff shock is small ex ante. In other words, a central banker say in 2005, putting very low probability on a trade war in 2025 would have announced a reaction function that looks very much like the one shown above. This result is proved formally in the appendix (Section (6.4)). The intuition is that the inflation increase when the tariff occurs has a small effect on inflation in the pre-tariff regime if price setters only put a small probability on the shock.

What about the Exchange Rate? The analysis above did not feature an explicit treatment of the exchange rate, but the exchange rate is adjusting implicitly in the background. We now resurface these effects and derive implications for the exchange rate.

Our model is a limit case of the classic model of an open economy with flexible exchange rates of Gali and Monacelli (2005), once we consider the limit in which all the goods produced by each country are perfect substitutes. Derivations in the appendix, show that we can write a continuous time version of the model of Gali and Monacelli

[13] See Woodford (2003).

(2005) where the following equation gives equilibrium domestic output:

$$y_t = \omega \frac{C}{Y} \left(c_t - \epsilon \left(p_{Ht} - p_t\right)\right) - \left(1 - \omega \frac{C}{Y}\right) \zeta \left(p_{Ht} - e_t\right).$$

In this equation there are four endogenous variables (in log-deviations from steady state): y_t is domestic output, p_{Ht} is the price of the home-produced good, p_t is the domestic CPI, and e_t is the nominal exchange rate. The coefficients in the equation are function of the following variables: C/Y, the steady state consumption to output ratio, ω, a parameter that captures home bias in consumption, ϵ, the elasticity of substitution between the home good and the foreign good, and ζ, the elasticity of substitution between goods produced in different countries.

Our model above corresponds to the case in which we take $\zeta \to \infty$. When that happens, there is perfectly elastic demand for the home good and the equation above is simply replaced by

$$p_{Ht} = e_t.$$

This is a simple version of a purchasing parity equation and tells us all we need to know about the exchange rate.

We then conclude that in our model the nominal exchange rate just tracks domestic inflation. If domestic inflation is higher due to the tariff, our currency will gradually depreciate. We do not see this as the most realistic prediction of the model at high frequency, as short run movements in the exchange rate are probably dominated by financial forces. But from the point of view of the relative prices that matter for allocations and inflationary pressures, it is possible that these high frequency currency fluctuations play a secondary role.

Seeing Through the Import Price Shock? A widely held tenet in the practice of central banking is that a shock like a tariff shock, that entails a mechanical increase in some prices, is a shock that requires the central bank to "see through it", in other words, to let the mechanical effect of the tariff in prices to affect the overall price level, but avoid larger increase in prices. To be fair, this is a rule of thumb which, as far as we know, does not correspond to any result in economic theory. However, it is useful to compare this practical precept to the optimal responses derived above.

A simple exercise is to take the cumulated effect of the shock on the price level

$$p = \lim_{t \to \infty} p_t = \int_0^\infty \pi_t dt$$

and compare it with the mechanical effect that would come from simply increasing import prices by τ, which, given the model structure, gives an effect proportional to the share of imported inputs in costs

$$s_M \tau.$$

Comparing these two quantities tells us whether the "see-through" principle holds exactly or approximately, given the model parameters.

In the model with sticky prices and flexible wages, the cumulated price response is derived in the proof of Proposition (2) and is

$$p = \frac{1}{\varepsilon s_L^2} \left(\frac{1}{1+\phi} + \eta \right) s_M \tau.$$

This expression can be in general smaller or larger than $s_M \tau$. For plausible parameter choices, and, in particular when ε is large enough, the principle seem to fail and the central bank wants less total inflation than see-through inflation. Notice that ε is the parameter that controls the welfare cost of inflation. The larger the parameter, the smaller the optimal cumulated inflation response. A somewhat surprising finding is that stickiness (the parameter θ) does not matter for this conclusion, which applies both in economies with low θ, which display a small and long-lasting increase in inflation, and in economies with high θ, which display larger but short-lived inflation.

However, the conclusion above is easily reversed when we turn to the model with sticky wages. With sticky wages, reaching a see-through outcome or a less-than-see-through outcome for prices becomes more costly, as it requires nominal wages to fall. In particular, the long-run price level response p always satisfies

$$p = s_L w + s_M (p + \tau) > s_M \tau$$

where w is the long-run effect on nominal wages. It is easy to show that the optimal policy will require more than see-through inflation if and only if

$$w > -\frac{s_M^2}{s_L} \tau,$$

that is if cumulated wage inflation is not too negative. In particular, if positive wage inflation is part of an optimal policy, as in the example of Figure(5), optimal inflation will always exceed see-through inflation.

Summing up, the see-through principle is easily violated in theory. In particular, when the economy displays both sticky prices and sticky wages, it seems violated in the direction that more than see-through inflation appears to be optimal. Whether from a quantitative point of view this result implies significant deviations from the see-through principle, is an open question for future work.

5 Conclusions

This paper examined the implications of tariffs for optimal monetary policy in an open economy with imported intermediate inputs. We show that tariffs act as cost-push shocks, introducing distortions that map exactly into labor wedge shocks in a closed economy.

This exact equivalence provides a powerful analytical lens through which to evaluate the central bank's trade-off between stabilizing inflation and supporting output. Our continuous-time framework allows us to characterize the optimal policy response sharply: a temporary accommodation of the shock, with inflation rising above target and output held above its new, lower natural level, before both variables gradually converge to the distorted steady state.

From a policy perspective, our analysis highlights the tension that tariffs create for central banks. The standard intuition that central banks should "see through" supply shocks does not hold unambiguously—optimal policy often calls for a transitory inflation overshoot to mitigate output losses. Moreover, we demonstrate that even when the steady state is distorted, the central bank's optimal response under commitment (including from the timeless perspective) supports a gradual transition.

Our analysis purposefully leaves out various channels, such as tariff and policy uncertainty, consumer confidence, and financial and currency instability. These may be important, but require a special and separate treatment.

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6 Appendix

6.1 Additional Derivations for Section (2)

First, we derive properties of $W(\tau)$, $A(\tau)$ defined as

$$W(\tau) = F(1, \mu(\tau)) - (1+\tau)\mu(\tau)$$

and

$$A(\tau) = F(1, \mu(\tau)) - \mu(\tau).$$

Differentiating W gives

$$W'(\tau) = -\mu(\tau), \quad W''(\tau) = -\mu'(\tau).$$

Differentiating A gives

$$A'(\tau) = F_M(1, \mu(\tau))\,\mu'(\tau) - \mu'(\tau) = \tau\mu'(\tau),$$

and

$$A''(\tau) = \tau\mu''(\tau) + \mu'(\tau).$$

Evaluated at $\tau = 0$, we have

$$A'(0) = 0, \quad A''(0) = \mu'(0).$$

Next, we can look at the effects on equilibrium welfare, defined as

$$\mathcal{U}(\tau) = U(A(\tau)L(\tau), L(\tau)).$$

The first derivative of $\mathcal{U}$ is

$$\mathcal{U}'(\tau) = U_C A'(\tau) L(\tau) + (U_C A(\tau) + U_L) L'(\tau)$$

or, using the labor supply condition $-U_L/U_C = W$,

$$\mathcal{U}'(\tau) = U_C \cdot \left[A'(\tau) L(\tau) + (A(\tau) - W(\tau)) L'(\tau)\right]. \tag{18}$$

Evaluating this expression at $\tau = 0$ we have $\mathcal{U}'(0) = 0$ because $A'(0) = 0$ and $A(0) - W(0) = 0$.

Differentiating (18) we get the second derivative of $\mathcal{U}$,

$$\begin{aligned}\mathcal{U}''(\tau) = U_C \cdot &\left[L(\tau) A''(\tau) + A'(\tau) L'(\tau) + \left(A'(\tau) - W'(\tau)\right) L'(\tau) + (A(\tau) - W(\tau)) L''(\tau)\right] + \\ &+ \frac{\partial}{\partial \tau}\left(U_C(A(\tau)L(\tau), L(\tau))\right)\left[A'(\tau) L(\tau) + (A(\tau) - W(\tau)) L'(\tau)\right].\end{aligned}$$

Evaluated at $\tau = 0$ the second line is zero and recalling that $A'(0) = 0$ we are left with

$$\begin{aligned}\mathcal{U}''(0) &= +U_C(\bar{C}, \bar{L})\left[\bar{L}A''(0) - W'(0)L'(0)\right] = \\ &= U_C(\bar{C}, \bar{L})\left[\bar{L}\mu'(0) + \mu(0)L'(0)\right],\end{aligned}$$

where bars denote variables at the zero tariff equilibrium. This gives us equation (7) in the text.

Finally, we prove $L'(0) < 0$. Labor supply satisfies

$$U_C(A(\tau)L(\tau), L(\tau))\,W(\tau) + U_L(A(\tau)L(\tau), L(\tau)) = 0.$$

Differentiating gives

$$(U_{CC}A + U_{CL})\,L' + U_C W' + (U_{LL}A + U_{LC})\,L' = 0$$

The expression

$$U_{CC}A + 2U_{CL} + U_{LL}A$$

is negative at $\tau = 0$ because it is the second-order condition of the optimization problem

$$\max_L U\,(A(0)L, L)$$

and $L(0)$ is the solution to this problem. Therefore, we have

$$L'(0) = -\frac{U_C W'(0)}{U_{CC}A + 2U_{CL} + U_{LL}A} < 0$$

because $W'(0)$.

We can express $\mu'\,(0)$ in terms of input shares and elasticities as follows. First, the proportional wage response satisfies

$$\frac{W'(0)}{W(0)} = -\frac{s_M}{s_L},$$

where s_M and s_L are the shares. Next, the proportional response of the input-to-labor ratio satisfies

$$\frac{\mu'(0)}{\mu(0)} = \eta\left(\frac{W'(0)}{W(0)} - 1\right) = -\eta\frac{1}{s_L},$$

where η is the elasticity of substitution between the two factors. If the preferences take the form

$$U\,(C, L) = \ln C - \frac{1}{1+\phi}L^{1+\phi},$$

we can then compute the labor response

$$\frac{L'(0)}{L(0)} = \frac{1}{1+\phi}\frac{W'(0)}{W(0)} = -\frac{1}{1+\phi}\frac{s_M}{s_L}.$$

Substituting we then have

$$\begin{aligned}\mathcal{U}''(0) = U_C\,(\bar{C}, \bar{L})\,\left[\bar{L}\mu'(0) + \mu(0)L'(0)\right] = \\ -\frac{1}{\bar{C}}\mu(0)\bar{L}\left[\eta\frac{1}{s_L} + \frac{1}{1+\phi}\frac{s_M}{s_L}\right] = \\ -\frac{s_M}{s_L}\left[\eta\frac{1}{s_L} + \frac{1}{1+\phi}\frac{s_M}{s_L}\right],\end{aligned}$$

where the last step uses $\bar{C}/\bar{Y} = \bar{W}\bar{L}/\bar{Y} = s_L$.

6.2 Proof of Proposition 2

Substituting $\omega_t = (1+\phi)\, l_t$, the per-period objective can be written as

$$-\frac{1}{2}\left[s_M\eta\left((1+\phi)\, l_t - \tau\right)^2 + (1+\phi)\, l_t^2 + \frac{\varepsilon}{\theta\,(\rho+\theta)}\pi_t^2\right].$$

Ignoring constant terms unaffected by policy this expression can be rewritten as

$$-\frac{1}{2}\left[\Psi_L\left(l_t - l^o\right)^2 + \Psi_P\pi_t^2\right],$$

where

$$\Psi_L = (1+\phi)\left(1+\eta s_M\,(1+\phi)\right),$$
$$\Psi_P = \frac{\varepsilon}{\theta\,(\rho+\theta)},$$

and

$$l^o = \frac{\eta s_M}{1+\eta s_M\,(1+\phi)}\tau.$$

We can also rewrite compactly the Phillips curve (8) as

$$\rho\pi_t = \kappa\left(l_t - \tilde{l}\right) + \dot{\pi}_t$$

where

$$\kappa = \theta\,(\rho+\theta)\, s_L\,(1+\phi)$$

and

$$\tilde{l} = -\frac{1}{1+\phi}\frac{s_M}{s_L}\tau.$$

The Hamiltonian for the central bank optimization problem can then be written compactly as

$$-\frac{1}{2}\left[\Psi_L\left(l_t - l^o\right)^2 + \Psi_P\pi_t^2\right] - \kappa\left(l_t - \tilde{l}\right)\nu_t + \pi_t\dot{\nu}_t$$

where ν_t denotes the Lagrange multiplier on the Phillips curve. The optimality conditions are

$$-\Psi_L\left(l_t - l^o\right) - \nu_t\kappa = 0, \tag{19}$$

$$-\Psi_P\pi_t + \dot{\nu}_t = 0. \tag{20}$$

Given that the initial π_0 is a free variable, we have the initial condition $\nu_0 = 0$ for the Lagrange multiplier ν_t. From (19), the initial value for l_t is then

$$l_0 = l^o.$$

Differentiating (19) we have

$$\Psi_L \dot{l}_t + \kappa \dot{\nu}_t = 0,$$

which, combined with (20), yields

$$-\Psi_P \pi_t + \dot{\nu}_t = 0$$

gives

$$\Psi_L \dot{l}_t + \kappa \Psi_P \pi_t = 0. \tag{21}$$

Differentiating the Phillips curve we have

$$\rho \dot{\pi}_t = \kappa \dot{l}_t + \ddot{\pi}_t$$

and substituting from (21) we obtain a second order ODE in π_t:

$$\rho \dot{\pi}_t = -\kappa^2 \frac{\Psi_P}{\Psi_L} \pi_t + \ddot{\pi}_t.$$

The solution takes the form

$$\pi_t = e^{-rt} \pi_0,$$

where r is the positive solution of the quadratic equation

$$r\left(r+\rho\right) = \kappa^2 \frac{\Psi_P}{\Psi_L} = \theta\left(\rho+\theta\right) \frac{s_L^2\left(1+\phi\right)\varepsilon}{1+\eta s_M\left(1+\phi\right)}. \tag{22}$$

The value of π_0 comes from the Phillips curve at date 0

$$\rho \pi_0 = \kappa\left(l^o - \tilde{l}\right) - r\pi_0$$

which gives

$$\pi_0 = \frac{\kappa}{r+\rho}\left(l^o - \tilde{l}\right) > 0. \tag{23}$$

The closed form solution for l_t is derived from (21) and the initial condition $l_0 = l^o$ and is

$$l_t - \tilde{l} = e^{-rt}\left(l^o - \tilde{l}\right).$$

The solution for output is

$$y_t = l_t + \eta s_M (\omega_t - \tau) = (1 + \eta s_M (1 + \phi)) l_t - \eta s_M \tau,$$

and the impact effect on output is

$$y_0 = (1 + \eta s_X (1 + \phi)) \frac{\eta s_M}{1 + \eta s_M (1 + \phi)} \tau - \eta s_M \tau = 0.$$

Cumulated inflation is

$$\int_0^\infty \pi_t dt = \int_0^\infty e^{-rt} \pi_0 = \frac{1}{r} \frac{\kappa}{r + \rho} \left(l^o - \tilde{l} \right) = \frac{\Psi_L}{\kappa \Psi_P} \left(l^o - \tilde{l} \right) = \frac{1 + \eta s_M (1 + \phi)}{\varepsilon s_L} \left(l^o - \tilde{l} \right)$$

where we used 22, 23 and the definitions of Ψ_L, Ψ_P, κ. Substituting

$$l^o - \tilde{l} = \frac{\eta s_M}{1 + \eta s_M (1 + \phi)} \tau + \frac{1}{1 + \phi} \frac{s_M}{s_L} \tau,$$

we get

$$\int_0^\infty \pi_t dt = \frac{1}{\varepsilon s_L^2} \left(\frac{1}{1 + \phi} + \eta \right) s_M \tau.$$

The total effect on the price level is larger than the direct effect $s_M \tau$ iff

$$\frac{1}{1 + \phi} + \eta > \varepsilon s_L^2.$$

6.3 Proof of Proposition (3)

6.3.1 Characterization of optimal path l_t

If we introduce sticky wages the model is characterized by two Phillips curves

$$\rho \pi_t = \Theta (s_L \omega_t + s_M \tau) + \dot{\pi}_t,$$

$$\rho \pi_{Wt} = \Theta_W [(1 + \phi) l_t - \omega_t] + \dot{\pi}_{Wt},$$

where

$$\Theta = \theta (\rho + \theta), \Theta_W = \theta_W (\rho + \theta_W)$$

and θ_W is the frequency of wage adjustment. The dynamics of the real wage are given by

$$\dot{\omega}_t = \pi_{Wt} - \pi_t.$$

A second-order approximation of the welfare function takes the form

$$-\frac{1}{2}\int_0^{\infty} e^{-\rho t}\left[\eta s_M\left(\omega_t-\tau\right)^2+\left(1+\phi\right)l_t^2+\Psi_P\pi_t^2+\Psi_W\pi_{Wt}^2\right]dt,$$

where

$$\Psi_P=\frac{\varepsilon}{\theta\left(\rho+\theta\right)},\Psi_W=\frac{\varepsilon_L}{\theta_W\left(\rho+\theta_W\right)},$$

and ε_L is the elasticity of substitution across different varieties of labor services.

The central bank problem is to choose $\{l_t\}$ to maximize the objective function, given $\omega_0=0$, subject to the constraints above.

The Hamiltonian can be written as follows

$$\begin{aligned}-\frac{1}{2}\left[\eta s_M\left(\omega_t-\tau\right)^2+\left(1+\phi\right)l_t^2+\Psi_P\pi_t^2+\Psi_W\pi_{Wt}^2\right]+\\ -\Theta_P\left(s_L\omega_t+s_M\tau\right)\nu_t+\pi_t\dot{\nu}_t+\\ -\Theta_W\left[\left(1+\phi\right)l_t-\omega_t\right]\nu_{Wt}+\pi_{Wt}\dot{\nu}_{Wt}+\\ \left(\pi_{Wt}-\pi_t\right)\mu_t-\rho\omega_t\mu_t+\omega_t\dot{\mu}_t,\end{aligned}$$

where ν_t,ν_{Wt},μ_t are Lagrange multipliers on the law of motions, respectively, of nominal prices, nominal wages, and of the real wage.

The optimality conditions are

$$-\left(1+\phi\right)l_t-\nu_{Wt}\Theta_W\left(1+\phi\right)=0, \tag{24}$$

$$-\Psi_P\pi_t+\dot{\nu}_t-\mu_t=0, \tag{25}$$

$$-\Psi_W\pi_{Wt}+\dot{\nu}_{Wt}+\mu_t=0, \tag{26}$$

$$-\eta s_M\left(\omega_t-\tau\right)-\Theta_P s_L\nu_t+\Theta_W\nu_{Wt}-\rho\mu_t+\dot{\mu}_t=0. \tag{27}$$

We have initial conditions $\nu_0=\nu_{W0}=0$ because the initial values π_0 and π_{W0} are free variables. Initial condition $\nu_{W0}=0$ implies $l_0=0$. The optimality condition for μ_0, given $\omega_0=0$ and $\nu_0=\nu_{W0}=0$, gives

$$\dot{\mu}_0=\rho\mu_0-\eta s_M\tau. \tag{28}$$

Differentiating (24) gives

$$\dot{l}_t+\dot{\nu}_{Wt}\Theta_W=0,$$

and substituting for $\dot{\nu}_{Wt}$ gives

$$\dot{l}_t+\left(\Psi_W\pi_{Wt}-\mu_t\right)\Theta_W=0. \tag{29}$$

Differentiating (27) gives

$$-\eta s_M \dot{\omega}_t - \Theta_P s_L \dot{v}_t + \Theta_W \dot{v}_{Wt} - \rho \dot{\mu}_t + \ddot{\mu}_t = 0,$$

and substituting for $\dot{\omega}_t, \dot{v}_t, \dot{v}_{Wt}$ gives the second order ODE for μ

$$-\eta s_X \left(\pi_{Wt} - \pi_t\right) - \Theta_P s_L \left(\Psi_P \pi_t + \mu_t\right) + \Theta_W \left(\Psi_W \pi_{Wt} - \mu_t\right) - \rho \dot{\mu}_t + \ddot{\mu}_t = 0$$

or

$$\left(\Theta_W \Psi_W - \eta s_X\right) \pi_{Wt} - \left(s_L \Theta_P \Psi_P - \eta s_X\right) \pi_t - \left(s_L \Theta_P + \Theta_W\right) \mu_t - \rho \dot{\mu}_t + \ddot{\mu}_t = 0.$$

The stable solution of this ODE satisfies

$$\dot{\mu}_t = r_1 \mu_t + \int_t^\infty e^{-r_2(s-t)} \left(\left(\Theta_W \Psi_W - \eta s_X\right) \pi_{Ws} - \left(s_L \Theta_P \Psi_P - \eta s_X\right) \pi_s\right) ds \qquad (30)$$

where r_1 and r_2 are the two roots of the quadratic equation

$$r\left(r - \rho\right) = s_L \Theta_P + \Theta_W,$$

ordered so that $r_2 > \rho > 0 > r_1$. Evaluating (30) at $t = 0$ and using (28) gives us the initial condition for μ

$$\mu_0 = \frac{1}{\rho - r_1} \left[\eta s_X \tau + \int_0^\infty e^{-r_2 t} \left[\left(\Theta_W \Psi_W - \eta s_X\right) \pi_{Wt} - \left(s_L \Theta_P \Psi_P - \eta s_X\right) \pi_t\right] dt\right].$$

$$\mu_t = e^{r_1 t} \mu_0 + \int_0^\infty H\left(t, s\right) \left[\left(\Theta_W \Psi_W - \eta s_X\right) \pi_{Wt} - \left(s_L \Theta_P \Psi_P - \eta s_X\right) \pi_t\right] ds \qquad (31)$$

where

$$H\left(t, s\right) = \frac{1}{r_2 - r_1} \left(e^{\min\{r_1(t-s), -r_2(s-t)\}} - e^{r_1 t - r_2 s}\right).$$

Steady state The new steady state is characterized by the terminal conditions for employment and real wages, equal to their flexible-price equilibrium levels:

$$\tilde{\omega} = -\frac{s_X}{s_L} \tau, \qquad \tilde{l} = -\frac{1}{1 + \phi} \frac{s_X}{s_L} \tau.$$

Inflation and the Lagrange multiplier μ converge to zero

$$\pi = \pi_W = \mu = 0.$$

However, the Lagrange multipliers on the inflation equations converge to positive values, reflecting the tension due to the distorted steady state, namely, we have

$$\nu_W \Theta_W = -l > 0,$$

$$\Theta_P s_L \nu = \eta \frac{s_X}{s_L} \tau + \Theta_W \nu_W > 0.$$

It is easy to check that all the equilibrium conditions and optimality conditions are satisfied given the variables and Lagrange multipliers above, with $\dot{\pi} = \dot{\pi}_W = \dot{\mu} = \dot{\nu} = \dot{\nu}_W = 0$.

Solution To solve for an equilibrium requires making a conjecture for l_t that satisfies the terminal conditions $l_0 = 0$ and $\lim_{t\to\infty} l_t = \tilde{l}$, solving for price and wage inflation, substitute in 31 and checking the optimality condition 29. To compute numerical solutions we use a finite difference method to solve for time derivatives and integrals, which yields a single linear equation in the vector representing the path l_t. The details are in the Jupyter notebook available online.

6.3.2 Proof of the results in the proposition

The initial condition $l_0 = 0$ and terminal condition $\lim_{t\to\infty} l_t = \tilde{l} < 0$ are proved in the characterization above. The results for output follow directly from the terminal conditions for l_t and ω_t and from $y_t = l_t + \eta s_M (\omega_t - \tau)$.

Combining the optimality conditions (25) and (26) yields

$$\Psi_P \pi_t + \Psi_W \pi_{Wt} = \dot{\nu}_t + \dot{\nu}_{Wt}.$$

Substituting $\pi_{Wt} = \pi_t + \dot{\omega}_t$ we obtain

$$(\Psi_P + \Psi_W) \pi_t = \dot{\nu}_t + \dot{\nu}_{Wt} - \Psi_W \dot{\omega}.$$

Given that the paths for $\nu_t, \nu_{Wt}, \omega_t$ are continuous and differentiable and satisfy the terminal conditions $\nu_0 = \nu_{W0} = \omega_0 = 0$ and

$$\lim_{t\to\infty} \nu_t > 0, \lim_{t\to\infty} \nu_{Wt} > 0, \lim_{t\to\infty} \omega_t < 0,$$

we have

$$(\Psi_P + \Psi_W) \int_0^\infty \pi_t dt > 0.$$

6.4 Optimal Response from the Timeless Perspective

Let $V(\pi)$ denote optimum welfare when the tariff τ is present and the central banker has committed to deliver the inflation rate π in the current period. The function V satisfies the Hamilton-Jacobi-Bellman equation

$$\rho V(\pi) = \max_{l} f\left(l, \pi; \tau\right) + V'(\pi)\left(\rho\pi - \kappa\left(l - \tilde{l}\right)\right),$$

where is the linear-quadratic expression

$$f\left(l, \pi\right) = -\frac{1}{2}\left[\Psi_L\left(l - l^o(\tau)\right)^2 + \Psi_P \pi^2\right]$$

and κ, Ψ_L, Ψ_P and l^o are defined as in the proof of Proposition (2).

The presence of a distorted flexible price allocation is captured by the fact that $f_l\left(\tilde{l}, 0\right) > 0$.

As shown in Proposition (2), the economy eventually reaches a long-run steady state with zero inflation in which

$$V'(0) = \frac{1}{\kappa} f_l\left(\tilde{l}, 0\right) > 0.$$

The function V is concave and has a maximum at $\pi = \pi_0 > 0$, which correspond to the optimal inflation rate on impact after the introduction of the tariff, as characterized in Proposition 2. That is, the problem with commitment and no past implies the choice of an initial inflation rate π_0, that satisfies

$$V'\left(\pi_0\right) = 0.$$

Consider now the policy problem in a situation preceding the introduction of tariffs. With Poisson arrival α there is a tariff shock and tariffs go from 0 to τ. The Phillips curve is now

$$\rho\pi = \kappa l + \alpha\left(\pi' - \pi\right) + \dot{\pi}$$

because agents are aware of the possible switch to a regime with tariffs and π' denotes the inflation on impact when the tariff is introduced. Also, the current allocation is undistorted so only l appears on the right-hand side.

Denote the value in the pre-tariff regime as $J(\pi)$. The Hamilton-Jacobi-Bellman equation is

$$\rho J(\pi) = \max_{l, \pi'} g\left(l, \pi\right) + J'(\pi)\left(\rho\pi - \kappa l - \alpha\left(\pi' - \pi\right)\right) + \alpha\left(V\left(\pi'\right) - J(\pi)\right),$$

where

$$g(l,\pi) = -\frac{1}{2}\left[\Psi_L l^2 + \Psi_P \pi^2\right].$$

This problem includes in every period: the commitment to satisfy the inflation promise π from the past, the optimal choice of real activity l, and the optimal choice of promised inflation in case the tariff shock were to hit, π'.

The optimal choice of π' yields the condition

$$J'(\pi) = V'\left(\pi'\right).$$

The remaining optimality conditions are optimality for l

$$g_l(l,\pi) = \kappa J'(\pi),$$

and the envelope condition

$$\rho J'(\pi) = g_\pi(l,\pi) + (\rho+\alpha) J'(\pi) - \alpha J'(\pi) + J''(\pi)\left(\rho\pi - \kappa l - \alpha\left(\pi' - \pi\right)\right),$$

which boils down to

$$g_\pi(l,\pi) + J''(\pi)\left(\rho\pi - \kappa l - \alpha\left(\pi' - \pi\right)\right) = 0.$$

If the economy remains in the pre-tariff state long enough, it is easy to show that it will converge to an allocation with zero inflation (from $g_\pi(l,\pi) = 0$), with the pair l, π' characterized by the two conditions

$$0 = \kappa l + \alpha \pi'$$

and

$$g_l(l,0) = \kappa V'\left(\pi'\right).$$

As $\alpha \to 0$ is then easy to show that the optimal solution converges to $l = 0$ and π' that satisfies

$$V'\left(\pi'\right) = 0.$$

This is the same condition that defines the inflation rate on impact in the commitment model with no past, we have showed that solution in Proposition (2) coincides with the timeless solution if $\alpha \to 0$.